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Algebraic and nested tree-valued processes

This thesis aims to study several tree-valued stochastic processes modeling the genealogical relationships within a population.
The first chapter is devoted to the infinite limit of the alpha model introduced by D. Ford. It is a one-parameter family of random binary trees with a fixed number of leaves which interpolates between the coalescent tree (also known as the Yule tree) and the branching tree (also known as the uniform tree). To construct the alpha-Ford models with an infinite number of leaves, we see them as elements of the space of binary algebraic measure trees and equip this space with the sample shape convergence introduced by W. Löhr and A. Winter. We show that the sequence of the alpha-Ford trees with an increasing number of leaves converges weakly. We then determine the annealed law of the statistics of subtree masses in the particular case of the Kingman algebraic measure tree. We also introduce through a well-posed martingale problem the alpha-Ford diffusion which generalizes the version of the Aldous diffusion constructed by W. Löhr, L. Mytnik and A. Winter. Finally, using that the alpha-Ford tree with infinitely many leaves is an invariant distribution of the alpha-Ford diffusion, we give a complete description of the annealed law of the statistics of subtree masses for any alpha-Ford tree through recursive relations on its moments.
In the second chapter, we are interested in the two-level version of two tree-valued resampling dynamics introduced by A. Greven, P. Pfaffelhuber and A. Winter. We first build the two-level tree-valued Moran dynamics as a stochastic process with values in the space of (ultra-)metric two-level measure spaces equipped with the two-level Gromov-weak topology, defined by R. Meizis. Under this model, a finite population of parasites divided in finitely many hosts undergoes resampling, both on the parasite and the host levels. Then, we show that the operator of this dynamics uniformly converge as the numbers of hosts and parasites both tend to infinity and that the martingale problem associated with the limit operator is well posed. The uniqueness of the solution results from a duality to the nested Kingman coalescent. We call the solution of the martingale problem the two-level tree-valued Fleming-Viot dynamics. Finally, we give formulas describing the evolution of the lengths of sampled subtrees under this dynamics.
The last chapter focuses on the space of algebraic two-level measure trees, which are the two-level analogues of the algebraic measure trees introduced by W. Löhr and A. Winter. Associating each algebraic (two-level measure) tree to the metric (two-level measure) space given by the distance arising from the distribution of branch points, we use the two-level Gromov-weak topology to define a metrizable topology. On the subspace of binary trees, we also introduce with the two-level sample shape convergence a more natural topology. We encode binary algebraic two-level measure trees with a triangulation of the circle together with a two-level measure on the circle line. Through this encoding, we prove that the two notions of topologies we defined on the subpace of binary algebraic two-level measure trees are equivalent and compact. We finish the chapter with a construction of the random algebraic two-level measure tree corresponding to the nested Kingman coalescent.