Mixed Consequence Logics

Quentin Blomet

In logic, a *logical consequence relation* is typically defined as a relation between propositions that ensures that truth is preserved from premises to conclusion in an argument. Wittgenstein (1922) defines the notion in the *Tractatus Logico-Philosophicus* (5.121) along the following lines:

> "The truth-grounds of *q* are contained in those of *p*: *p* follows from *q*."

Tarski (1936), building on Carnap's reflections, presents the concept with a slight variation, focusing on the incompatibility between the truth of the premises and the falsity of the conclusion:

> "From an intuitive standpoint, it can never happen that the class *K* consists only of true sentences while the sentence *X* is false."

Since truth and non-falsity coincide in classical logic, these two definitions are effectively equivalent within a classical framework. However, this reliance on non-falsity opens the door to alternative definitions of logical consequence. By introducing three truth values, 0, 1/2, 1, the set of truth values can be partitioned in more than one way. Non-falsity corresponding now to the set {1/2, 1} and non-truth to {0, 1/2}. One might therefore define a consequence relation preserving only the value 1, thereby adopting a *strict* approach regarding the values preserved from premises to conclusion. The difference with the classical case appears clearly when the consequence relation is defined along Tarski's lines, ensuring that it is impossible for the premises to take the value 1, and the conclusion the value 0 *or* 1/2. Alternatively, one could want to adopt a *tolerant* approach, requiring the values in {1/2, 1} to be preserved from premises to conclusion for an inference to be valid. These considerations suggest more generally that we might aim to define logical consequence as the necessary preservation of a specific *set of designated values* from premises to conclusion. Validity, then, would capture the idea that whenever the premises are assigned a value in this set, the conclusion should also be assigned a value in this set.

A natural extension of this approach involves applying different standards to evaluate the premises and the conclusion. One might want to move from the non-falsity of the premises to the truth of the conclusion, or vice versa. That is, in the three-valued case, from {1/2, 1} to {1}, or from {1} to {1/2, 1}; leading to an even more permissive notion of logical consequence. This approach not only partitions truth values in multiple ways but also allows premises and conclusions to be associated with different partitions. Validity, then, would express the idea that whenever the premises are assigned a value in the first set, the conclusion should be assigned a value in the second set. The consequence relations that ensure this are known as *mixed consequence relations*, and the logics based on these relations are referred to as *mixed consequence logics*. The topic of this dissertation is the investigation of these mixed consequence logics in a three-valued setting.

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