Résume de la thèse « Inégalités systoliques en géométrie métrique et de contact »

Anglais :

On a closed contact 2n+1-manifold, the systolic ratio of a contact form is the ratio between the n+1 -th power of the minimum of the periods of closed Reeb orbits by the contact volume. A family of contact forms C satisfies a systolic inequality if the systolic ratio is bounded on C. This generalizes the well-studied setting of systolic inequalities in Riemannian geometry. It is known that there is no systolic inequality valid for all contact forms on a fixed contact manifold. The main result of this thesis is that, on an oriented closed three-dimensional manifold endowed with an almost-free circle action with non-zero Euler number, the class of contact forms invariant under this action satisfies a systolic inequality. Although this result does not give any optimal bound, we prove a sharp systolic inequality under more restrictive assumptions, namely for tight invariant contact forms on circle bundles over the two-sphere. In that case, we fully characterize when equality is reached. We then give applications of this last result: a sharp systolic inequality for the contractible Reeb orbits on some lens spaces, a sharp systolic inequality for Finsler metrics of revolution on the two-sphere and the validity of a conjecture of Viterbo for convex domains invariant under a symplectic circle action.