Université
franco-allemande
Deutsch-Französische
Hochschule

Entanglement and Quantumness
- New Numerical Approaches -

The main topic of this compilation thesis is the investigation of multipartite entanglement of finite dimensional systems. We developed a numerical algorithm that detects if a multipartite state is entangled or separable in a finite number of steps of a semi-definite optimization task. This method is an extension of previously known semi-definite methods, which are inconclusive when the state is separable. In our case, if the state is separable, an explicit decomposition into a mixture of separable states can be extracted. This was achieved by mapping the entanglement problem onto the mathematically well studied truncated moment problem. Additionally, a new way of writing the partially transposed state for symmetric multi-qubit states was developed which simplifies many results previously known in entanglement theory. This new way of writing the partial transpose criterion unifies different interpretations and alternative formulations of the partial transpose criterion and it is also a part in the aforementioned semi-definite algorithm.

The geometric properties of entangled symmetric states of two qubits were studied in detail: We could answer the question of how far a given pure state is from the convex hull of symmetric separable states, as measured by the Hilbert-Schmidt distance, by giving an explicit formula for the closest separable symmetric state. For mixed states we could provide a numerical upper and analytical lower bound for this distance. For a larger number of qubits we investigated the ball of absolutely classical states, i.e. symmetric multi-qubit states that stay separable under any unitary transformation. We found an analytical lower bound for the radius of this ball around the maximally mixed symmetric state and gave a numerical upper bound on this radius, by searching for an entangled state as close as possible to the maximally mixed symmetric state.

The tensor representation of a symmetric multi-qubit state, or spin-j state, allowed us to study entanglement properties based on the spectrum of the tensor via tensor eigenvalues. The definiteness of this tensor relates to the entanglement of the state it represents and, hence, the smallest tensor eigenvalue can be used to detect entanglement. However, the tensor eigenvalues are more difficult to determine than the familiar matrix eigenvalues which made the investigation computationally more challenging. The relationship between the value of the smallest tensor eigenvalue and the amount of entanglement in the state was also investigated. It turned out that they are strongly correlated for small system sizes, i.e. for up to six qubits. However, to investigate this correlation we needed an independent way to gauge the amount of entanglement of a state and in order to do so we improved existing numerical methods to determine the distance of a state to the set of separable symmetric states, using a combination of linear and quadratic programming. The tensor representation of symmetric multi-qubit states was also used to formally define a new tensor class of regularly decomposable tensors that corresponds to the set of separable symmetric multi-qubit states.