Université
franco-allemande
Deutsch-Französische
Hochschule

Stochastic models for collective motions of populations

Laure Pédèches

In this thesis, stochastic dynamics modelling collective motions of populations, one of the most mysterious type of biological phenomena, are considered. They can be observed amongst flocks of birds and schools of fish, but also in some colonies of bacteria, herds of cattle or human populations. This kind of phenomena also appears in various other fields such as finance, linguistics or robotics.

We study the dynamics of a system of N particle-like individuals, focusing on two kinds of asymptotic behaviours.
On the one hand, ergodic properties in long time: existence of an invariant probability measure via Lyapunov functionals, rate of convergence of the transition semi-group towards this probability measure. Also at the core of our work is the notion of flocking: we defined it as the fact that a set of individuals reaches a consensus without a hierarchical structure; from a mathematical perspective, this corresponds to the alignment of the velocities and to the swarming of the individuals.
On the other hand, we study the propagation of chaos phenomenon when the number N of particles goes to infinity: the behaviours of the different individuals become asymptotically independent.

The Cucker-Smale model, a mean-field kinetic deterministic model for a population without hierarchical structure is our starting point. The interaction between two particles depends on their "communication rate", which is a function of their relative distance and decreases at polynomial speed.

In the first chapter, we study the asymptotic behaviours of a randomly perturbed Cucker-Smale model, and of some of its variants.

Chapter 2 presents various definitions for flocking in a stochastic setting: several stochastic models, corresponding to different noises – representing for instance a noisy environment, the "free will" of each individual or a scrambled transmission – are presented and studied in conjunction with these notions.

The third chapter is built upon the cluster expansion method, a technique from statistical mechanics. We prove the exponential ergodicity for a class of non-Markovian process with non-regular drift, and we apply those results to perturbations of the Ornstein-Uhlenbeck process.

In the final part, the focus shifts to Keller and Segel 2-D parabolic-elliptic model for chemotaxis, and in particular to the mean-field particle system that can be derived from it. We prove the existence of a solution, unique in some way, by determining the possible types of collisions between particles, thanks to comparisons with Bessel processes and Dirichlet form theory.